1 RCMSP

The Resource Cconstrained Modulo Scheduling Problem is a 1-periodic scheduling problem where λ corresponds to the cycle length. The goal of this model is to find a schedule for the first iteration of each task that has to be repeated each λ time. In our problem *lambda* is a fixed problem parameter rather than a variable to avoid non-linearity. There is two temporal parameters, t the absolute time and $\tau = t \ mod \ \lambda$ $(t = \tau + k * \lambda)$

The initial model is described as following: the variable x_i^t is equal to 1 if the first iteration of the task i is scheduled at time t. Using the Dantzig-Wolfe decomposition, a new pattern variable z_l^τ is introduced equal to 1 if the pattern l is scheduled at time τ (a pattern is a set of task at a given τ). a_i^l is a problem parameter equal to 1 if the task i is included in the pattern l, so $A = [a_i^l]_{i,l}$ corresponds to the matrix of columns.

Due to the configuration of the Dantzig-Wolfe decomposition, there would be λ sub problem generating at most λ pattern at each iteration, hence the temporal dimension of the patterns.

$$min \sum_{i=1}^{n} w_i \sum_{t=0}^{T-1} t x_i^t$$

subject to

$$\sum_{t=0}^{T-1} x_i^t \ = \ 1 \quad i \in \{1, ..., n\} \tag{1}$$

$$\sum_{l \in R} a_i^l (\sum_{\tau=0}^{\lambda-1} z_l^\tau) \ = \ 1 \quad i \in \{1, ..., n\} \tag{2}$$

$$\sum_{l \in R} z_l^\tau \ \leq \ 1 \quad \tau \in \{0, ..., \lambda - 1\} \tag{3}$$

$$\sum_{t=0}^{T-1} t x_i^t + \theta_i^j - \lambda \omega_i^j \ \leq \ \sum_{t=0}^{T-1} t x_j^t \quad (i, j) \in E \tag{4}$$

$$\sum_{l \in R} a_i^l z_l^\tau \ = \ \sum_{k=0}^{K-1} x_i^{\tau + k\lambda} \quad \forall i \in \{1, ..., n\}, \ \forall \tau \in \{0, ..., \lambda - 1\} \tag{5}$$

$$x_i^t \in \{0, \ 1\} \quad \forall i \in \{1, ..., n\}, \ \forall t \in \{0, ..., T - 1\}$$

$$z_l^\tau \ \in \ \{0, \ 1\} \quad \forall l \in R, \ \forall \tau \in \{0, ..., \lambda - 1\}$$

The constraint 4 corresponds to the precedence constraint, while the constraint 5 corresponds to the equivalence between the variables. The constraint 1 states that each task must be scheduled at one time, the constraint 2 states that each task must be associated to only one pattern and finally the constraint 3 states that at most one pattern can be scheduled at a given $\tau = t \ mod \ \lambda$.

The dual constraints associated with the pattern variables:

$$\sum_{i=1}^{n} a_i^l \rho_i + \beta_\tau + \sum_{i=1}^{n} a_i^l \gamma_i^\tau \leq 0, \quad \forall l \ \in \ R, \ \tau \in \{0, ..., \lambda - 1\}$$

ρ_i is the dual variable associated with the constraint 2, β_τ the dual variable associated with 3 and γ_i^τ associated with 5.

These constraints lead to λ knapsack subproblems with resource constraints:

$$min \ -\sum_{i=1}^{n} a_i \rho_i - \beta_\tau - \sum_{i=1}^{n} a_i^l \gamma_i^\tau$$

subject to

$$\sum_{i=1}^{n} a_i b_i^s \leq B_s, \quad \forall s \in \{1, ..., m\}$$

$$a_i \in \{0, \ 1\}, \quad \forall i \in \{1, ..., n\}$$

Each subproblem is supposed to generate a pattern only for its associated τ, however it is possible to simply add the pattern generated to all τ, which corresponds to what has been done in my code.

In order to determine an initial feasible subset, it is better to first solve a relaxed version of the master problem by introducing a set of new variables c_τ.

$$min \sum_{\tau=0}^{\lambda-1} c_\tau$$

subject to

$$\sum_{t=0}^{T-1} x_i^t \ = \ 1 \quad i \in \{1, ..., n\} \tag{6}$$

$$\sum_{l \in R} a_i^l (\sum_{\tau=0}^{\lambda-1} z_l^\tau) \ = \ 1 \quad i \in \{1, ..., n\} \tag{7}$$

$$\sum_{l \in R} z_l^\tau - c_\tau \ \leq \ 1 \quad \tau \in \{0, ..., \lambda - 1\} \tag{8}$$

$$\sum_{t=0}^{T-1} t x_i^t + \theta_i^j - \lambda \omega_i^j \ \leq \ \sum_{t=0}^{T-1} t x_j^t \quad (i, j) \in E \tag{9}$$

$$\sum_{l \in R} a_i^l z_l^\tau \; = \; \sum_{k=0}^{K-1} x_i^{\tau + k\lambda} \quad \forall i \in \{1, ..., n\},\ \forall \tau \in \{0, ..., \lambda - 1\} \tag{10}$$

$$x_i^t \in \{0,\ 1\} \quad \forall i \in \{1, ..., n\},\ \forall t \in \{0, ..., T-1\}$$

$$z_l^\tau \;\in\; \{0,\ 1\} \quad \forall l \in R,\ \forall \tau \in \{0, ..., \lambda - 1\}$$

The convexity constraints 8 are relaxed to generate new columns until this same constraints are saturated, which means that the set of columns is now feasible for the true master problem. The identity matrix can be used as an initial subset for this new model. The model will generates a true feasible subset which will then be used as an initial subset in the true model. The relaxed model corresponds to the model implemented in the code.